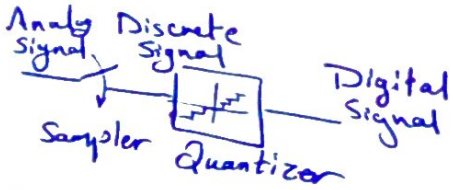
Digital Control (Session 2)

* Analog / Digital Conversion

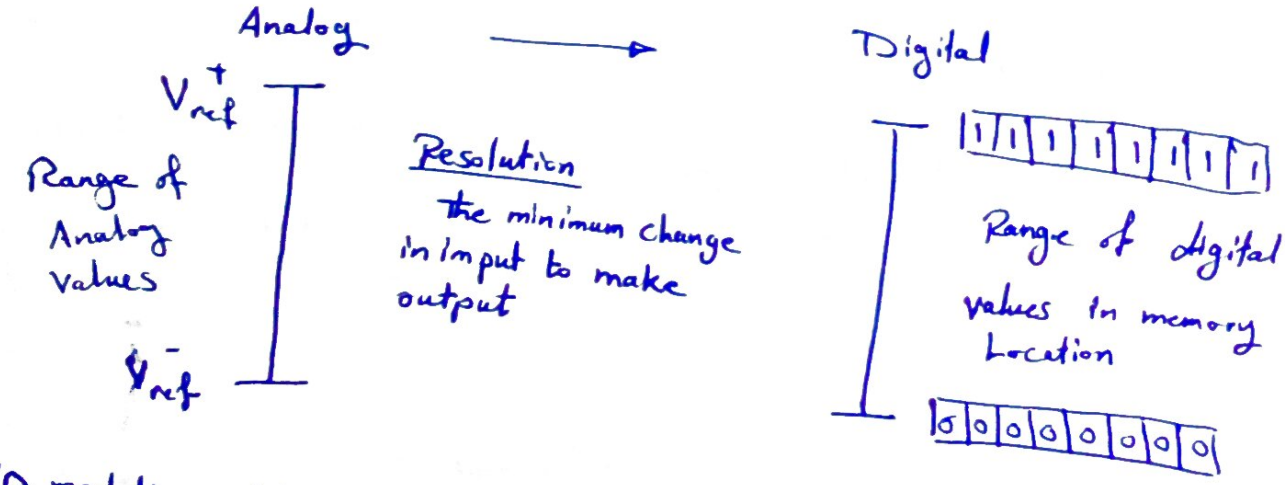
- Digital Systems require digital input to deal with and generate (produce) digital output. the Conversion from Analog world to digital is necessary
- Our analysis is applied to discrete signals (continous in amplitude, discrete in time). In practice, using a digital platform to implement digital Controller requires signals to be digital (Discrete in both time and amplitude).

is two stages



- The value of signal at each Sample is stored in finite length word (memory location in the Computer System)

* Analog / Digital Conversion equations



A/D resolution: determines the analog value required to increment the value in register by 1

A/D resolution: determined by width of register (n-bits)

$$\text{Resolution} = \frac{V_{ref}^{+} - V_{ref}^{-}}{2^{n} - 1}$$

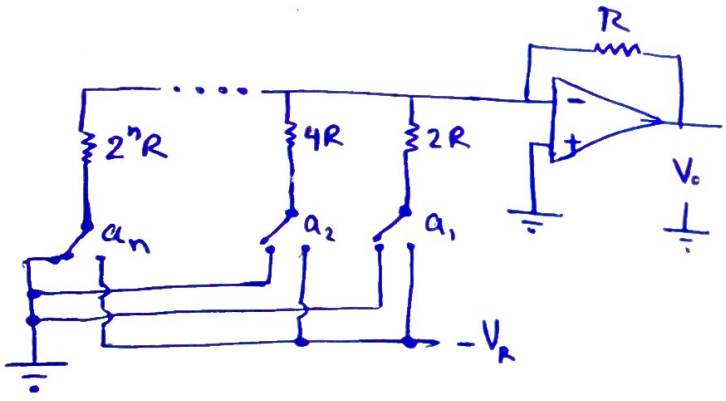
↳ Larger n means higher accuracy for A/D conversion

$$\text{Analog Value} = \text{Resolution} * \text{Digital Value}$$

* DAC : Digital to Analog Converters

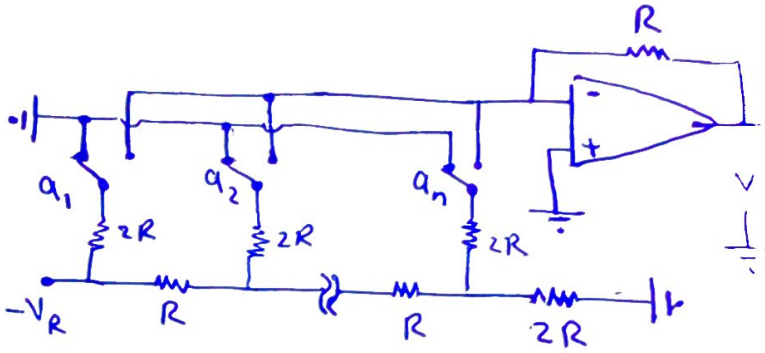
$$V_o = V_R [a_1 2^{-1} + a_2 2^{-2} + \cdots + a_n 2^{-n}]$$

where : V_R : reference voltage
V_o : Analog Output Value
$a_1 - a_n$: Digital Values of bits
a_1 : MSB , a_n : LSB



(Weighted Resitor DAC)

- Requires large number of different values of resistors (Disadvantage)

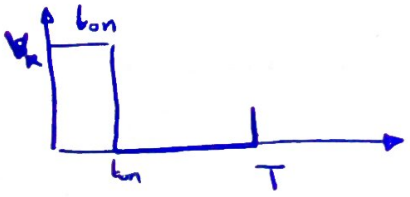


(Inverted R-2R ladeler)

Requires Only two different values of resistors

* DAC vs. PWM,

PWM : Pulse Width Modulation



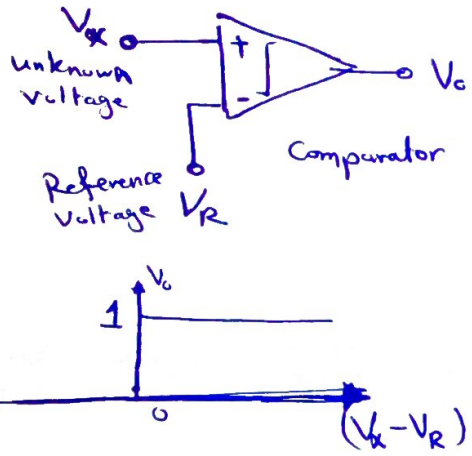
The output has two cases 0 or V_R-volt (Digital) but t_{on} is changed

$$V_{avg} = \frac{1}{T} \int_0^T v(t)\,dt = \frac{1}{T} \int_0^{t_{on}} V_R\,dt + 0$$

$$\boxed{V_{avg} = \frac{t_{on}}{T} V_R = k V_R} \quad , \quad \boxed{k : \text{duty cycle } (0-1)}$$

* ADC : Analog to Digital Converters

Basic Idea : Compare unknown voltage with Reference voltage to determine level (Binary levels)



V_R is varied to determine which of 2^n possible binary words is closest to V_x.

V_R can assume 2^n different values of the form

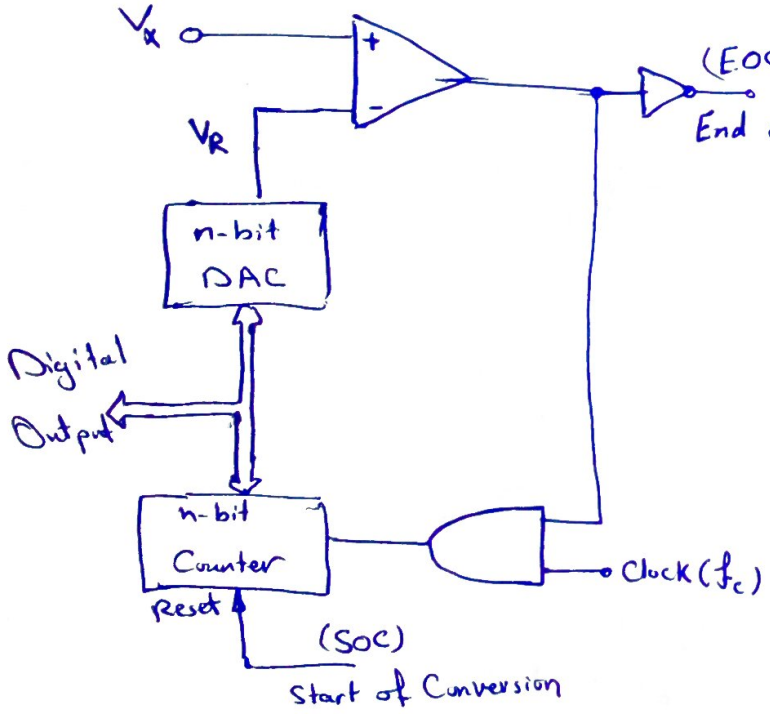
$$V_R = V_r \sum_{i=1}^{n} A_i 2^{-i}$$

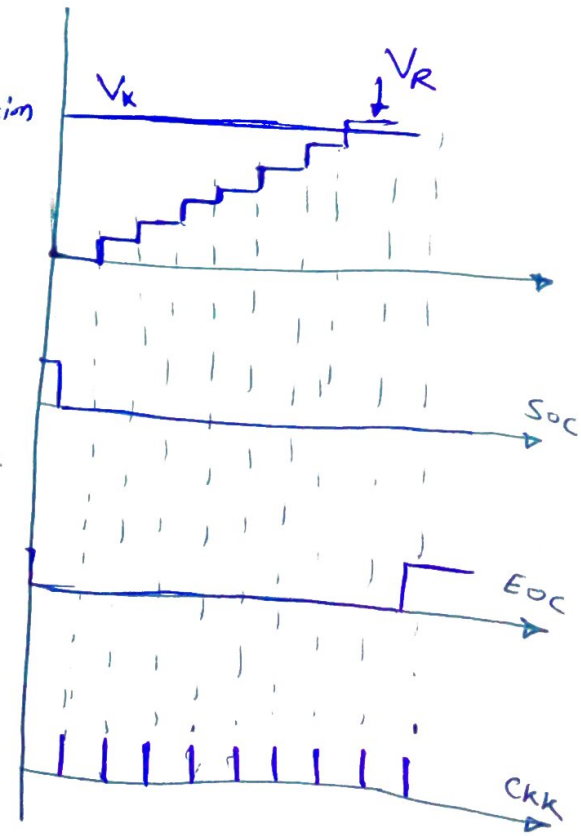
V_r : DC reference Voltage

A_i : Binary Coeffecients

$\therefore$ A_i are choosed so that $\text{error} = |V_x - V_R| = \left| V_x - V_r \sum_{i=1}^{n} A_i 2^{-i} \right|$ be minimal

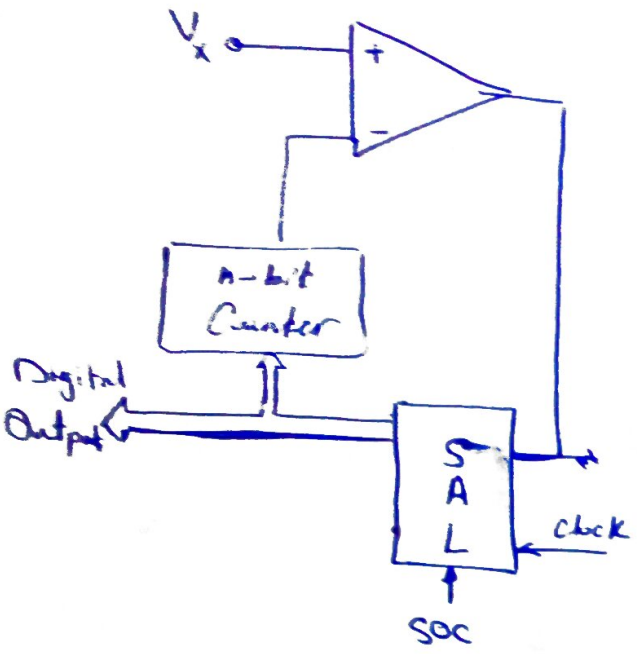
① Counter Ramp Converter



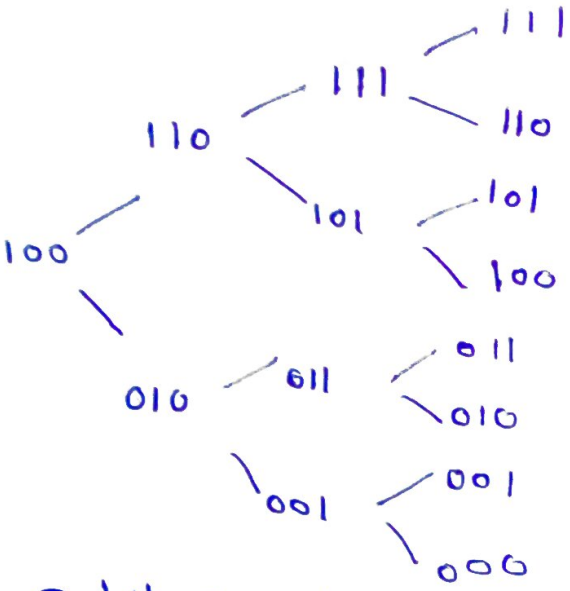


$\therefore$ Requires multiple clocks to do Conversion

(2) Successive Approximation ADC

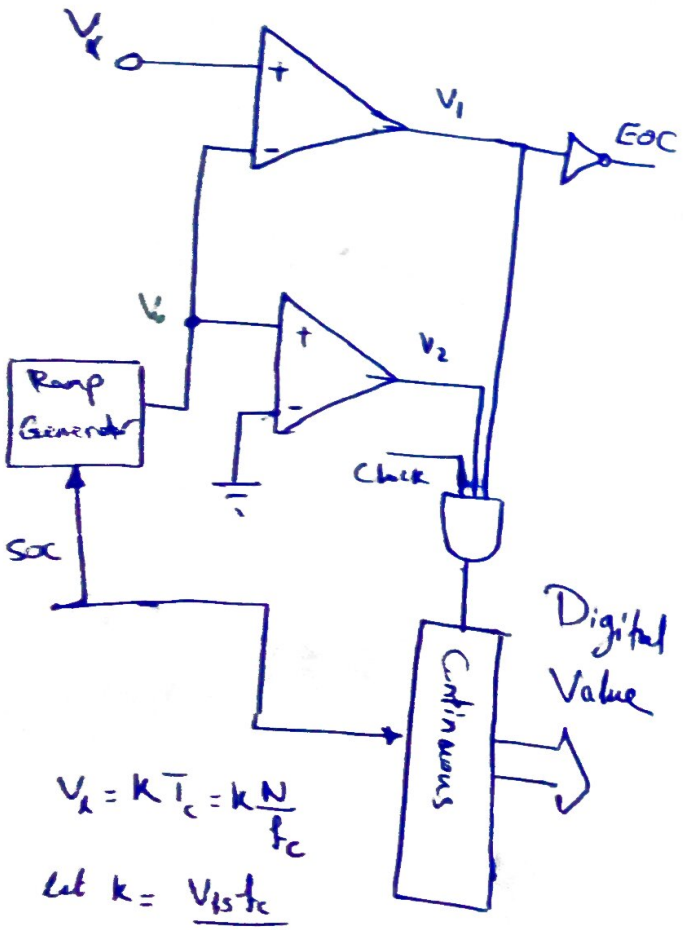


↳ requires only n-clock to do Conversion



3-bits successive Approximation DAC

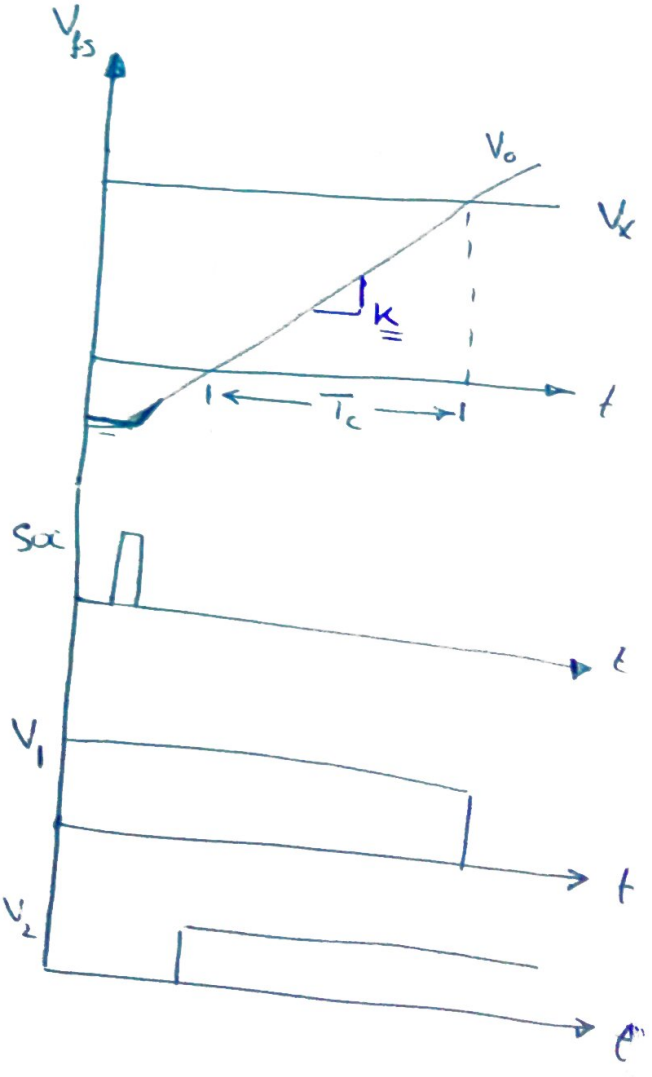
(3) Single Ramp Converter



$V_x = K\bar{T}_c = k\frac{N}{f_c}$

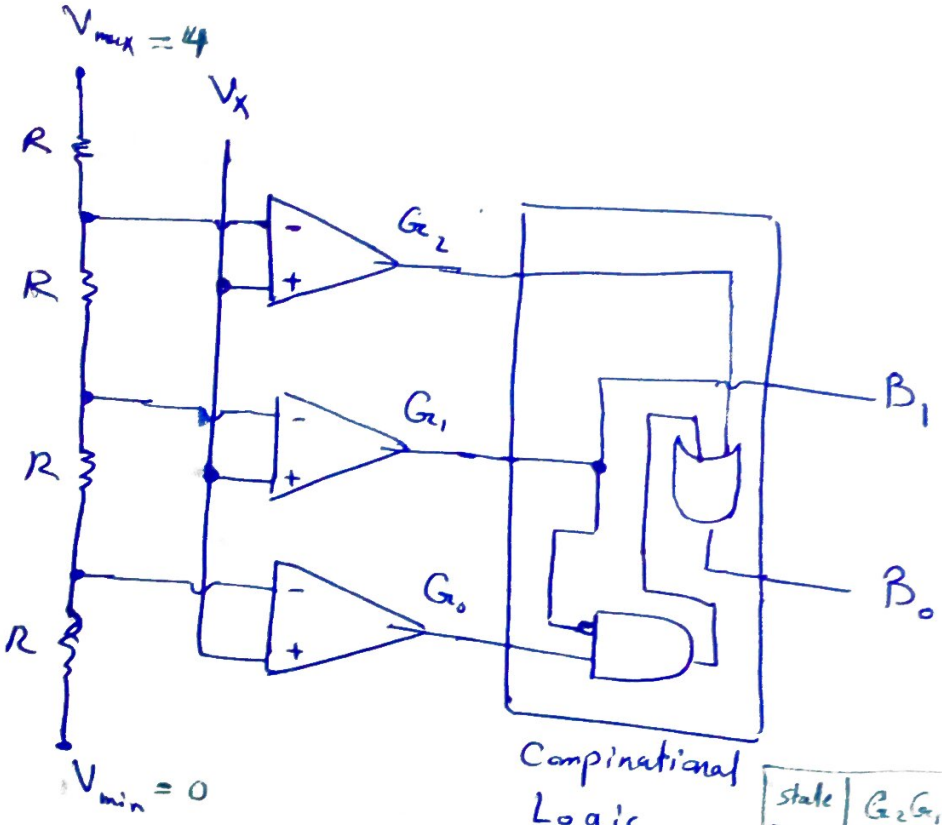
Let $k = \frac{V_{fs} f_c}{2^n}$

$V_x = \frac{V_{fs} N}{2^n}$



3 Parallel Converter (Flash)

- use additional Hardware to perform Parallel rather than Serial Conversion
- Fastest Converter
- V_x is Simultaneously Compared to different reference values



2-Bit Converter

state	$G_2G_1G_0$	B_1B_0	Voltage
0	000	00	0 - 1
1	001	01	1 - 2
2	011	10	2 - 3
3	111	11	3 - 4

$B_0 = G_0\overline{G_1} + G_2$

$B_1 = G_1$

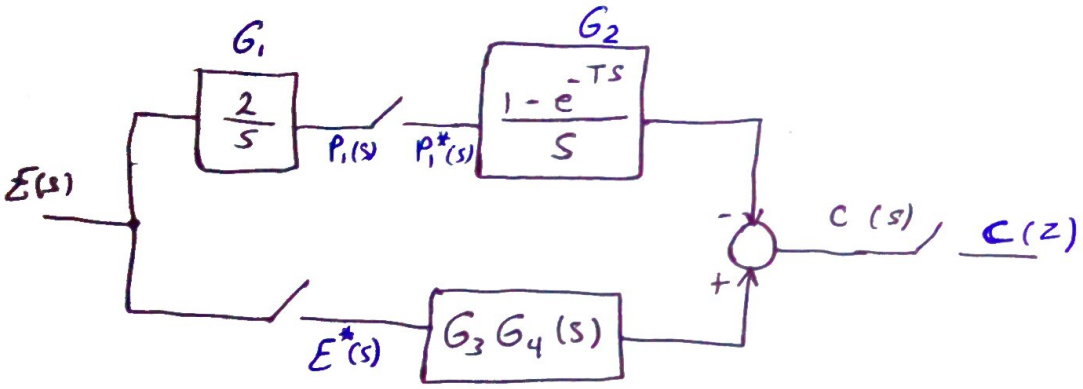
Another Types of ADC :

- Tracking ADC
- Dual Ramp Converter

* Reference :

C. L. Phillips, H. Troy Nagle, "Digital Control System Analysis and Design", 3rd edition - Chapter 3 - Sections 3.8, 3.9

Section 3

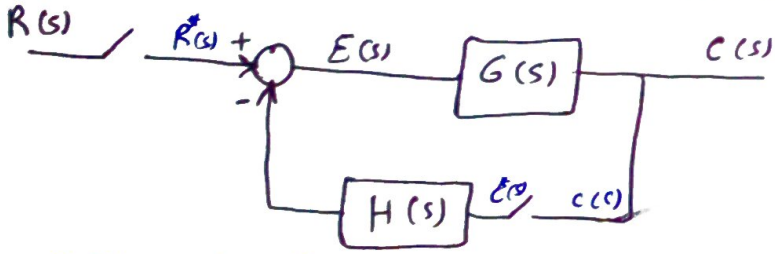


$$C(s) = E^*(s)\, G_3 G_4(s) - G_2 P_1^*(s)$$

$$P_1(s) = E(s)\, G_1(s) \longrightarrow P_1^*(s) = \overline{G_1(s)\, E(s)}^*$$

$$C(s) = E^*(s)\, G_3 G_4(s) - G_2(s)\, \overline{E G_1}^*(s)$$

$$C^*(s) = E^*(s)\, \overline{G_3 G_4}^*(s) - G_2^*(s)\, \overline{E G_1}^*(s)$$



$$C(s) = G(s)\, E(s)$$

$$E(s) = R^*(s) - C^*(s)\, H(s)$$

$$C(s) = G(s)\left[R^*(s) - C^*(s)\, H(s) \right]$$

$$C^*(s) = G^*(s)\, R^*(s) - C^*(s)\, \overline{GH}^*(s)$$

$$\left(1 + \overline{GH}^*(s)\right) C^*(s) = G^*(s)\, R^*(s)$$

$$\frac{C^*(s)}{R^*(s)} = \frac{G^*(s)}{1 + \overline{GH}^*(s)}$$